# Integrating Computing Techniques for Parametric Sensitivity Analysis in Control System Optimization

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*--***ABSTRACT***--*

**This research paper focuses on developing a mathematical model to assess the parametric sensitivities of an operational induction motor equipped with a solid hollow ferromagnetic rotor. The analytical approach involves integrating principles from both electromagnetic circuit theory and electromagnetic field theory. The derivation of the parametric sensitivity matrix entails constructing differential equations, specifically the first variation equations derived from the motor's electromechanical state equations.**

**The study further extends its analysis by conducting computer simulations to explore the parametric sensitivities related to the material composition of the rotor body and the characteristics of the input signal. The results of these simulations provide valuable insights into how alterations in these parameters impact the motor's performance. By combining theoretical foundations with practical computational simulations, this research offers a comprehensive understanding of the interplay between various factors influencing the behavior of the induction motor with a solid hollow ferromagnetic rotor.**

Keywords - **computing techniques, control systems, electromagnetic field, parametric sensitivity, solid ferromagnetic rotor.**



# **1. INTRODUCTION**

 $T$ he availability of a sensitivity matrix holds paramount importance in the context of optimizing the design of sophisticated control systems. The concept of sensitivity function, elucidating the impact on sensitivity characteristics, as articulated by Tamas Gal and H. J. Greenberg in 1997, has been a staple in circuit theory and has recently found successful application in the theory of electromagnetic fields, as evidenced by works such as those by Ali M. Niknejad in 2007 and Henry W. Ott in 2009.

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In both circuit theory and the electromagnetic field theory, the absolute sensitivity function is termed parametric sensitivity, underscoring its distinction from sensitivity to initial conditions, a phenomenon extensively explored by Henry W. Ott in 2009. The practical alignment of sensitivity to initial conditions with parametric sensitivity is evident when considering the initial condition column in the system state's differential equations, denoted as the constant parameter, as articulated by Steven G. Krantz in 2014. This alignment is possible since initial conditions are unrelated to unknown quantities and represent specified values of these unknowns at the moment  $t=+0$ , as expounded by George F. Simmons in 2016.The control system plays a particular part in technological progress (Ralph Morrison,2016)[1,3,9].

The advancement of mathematical methodologies in control systems intricately involves the formulation of mathematical models for elemental foundations. Primarily,

this encompasses the creation of elements responsible for converting electrical signals within the system into various forms of motion, such as mechanical, translational, rotary, oscillatory, and others, as elucidated by Andrei I. Subbotin in 2013[2,7,15]. The precision, quality, and reliability of control systems are inherently contingent upon the accurate execution of directives by electromechanical instruments. Among these instruments, actuating induction motors hold a distinctive position, as highlighted by Ove Glenberg in 2010.

The scope of this investigation encompasses actuating induction motors, recognized as the most intricate components within the control system[4,8,10], as expounded by Austin Hughes and Bill Drury in 2013, and discussed by Bahram Amin in 2010. Three fundamental configurations of actuating induction motors are identified, namely those featuring a squirrel-cage rotor, a hollow drum with continuous ferromagnetic rotor [5], and a variant with a non-magnetic rotor[6]. The construction of a motor with a cage rotor mirrors that of a typical industrial motor, as outlined by B.A. Behrend in 2016[11].

Alternatively, a motor with a solid non-magnetic rotor encased in a hollow drum exhibits both internal and external stators. The excitation and control windings are situated on the external stator, as noted by E.P. Wohlfarth in 1986[12]. In contrast, a hollow ferromagnetic rotor necessitates no windings on the external stator due to the relatively low magnetic resistance, as underscored by Kenneth L. Kaiser in 2004 and Helmut Kronmuller, and Manfred Fahnle in 2009[13]. This results in a significantly simplified motor construction.

The culmination of this study involves the construction of mathematical models for parametric sensitivity, representing the final and most intricate phase not only in the analysis of control systems but also in the examination of an unbounded object. This construction is contingent upon the previous stages of analysis having viable solutions according to the general theory of nonlinear differential equations, as emphasized by Paul Sherz and Simon Monk in 2013[14].

Advancement in the computational methodologies for determining the transient parametric sensitivity of an actuating induction motor with a solid hollow ferromagnetic rotor necessitated the resolution of the primary challenges outlined below:

1. Development of an algorithm for calculating parametric sensitivity concerning the vector potential of a quasistationary electromagnetic field with respect to constant parameters within a two-dimensional nonlinear ferromagnetic medium.

2. Integration of the system of variational differential equations characterizing both the nonlinear stator circuit and the nonlinear ferromagnetic rotor medium.

3. Formulation of a comprehensive system of differential equations, encompassing both state and variational equations for parametric sensitivity, and subsequent joint integration of this system over the specified time interval.

These endeavors were essential to achieving the overarching goal of enhancing the computational methods associated with transient parametric sensitivity analysis in actuating induction motors with solid hollow ferromagnetic rotors[16,17,18].

#### **1.2 RESEARCH NOVELTIES**

A matrix characterizing the parametric sensitivity of potential, vectors, integral quantities of the electromagnetic field, and mechanical variables with respect to the constant parameters of an actuating induction motor featuring a solid hollow ferromagnetic rotor was systematically developed. This matrix serves as a comprehensive representation of the intricate interdependencies between the specified variables and the constant parameters governing the motor's behavior.

Concurrently, an algorithm designed for the purpose of calculating the aforementioned matrix of parametric sensitivity for the actuating induction motor with a solid hollow ferromagnetic rotor was established. This algorithm is structured to efficiently compute the sensitivity values with respect to the chosen constant parameters, thereby facilitating a detailed and nuanced analysis of the motor's response to variations in these key factors.

# **2. RESEARCH METHODOLOGY**

The methodology employed in theoretical investigations relies on the formulation and analysis of nonlinear partial differential equations governing a quasistationary electromagnetic field within a continuous medium. This encompasses nonlinear differential equations describing electromagnetic circuits in terms of ordinary derivatives, as

well as nonlinear differential equations governing mechanical motion [18,19]. Additionally, numerical techniques for solving differential equations and methods for addressing nonlinear algebraic equations play pivotal roles in advancing these theoretical studies.

#### **2.1 SIGNIFICANCE OF THE OBTAINED RESULTS**

The newly created computer program is designed to compute the transient parametric sensitivity matrix of an actuating induction motor, particularly one equipped with a solid hollow ferromagnetic rotor. This matrix is calculated with respect to selected constant parameters, providing a comprehensive analysis of the motor's dynamic response. The program is versatile and can be applied in the examination of individual motors during various stages of development or performance assessment. Furthermore, its adaptability extends to functioning as a module within a broader program aimed at determining the parametric sensitivity of the entire control system governing such motors [19]. This modular integration enhances the program's utility by facilitating a holistic evaluation of the system's responsiveness to different parameter variations [20].

# **2.1 CONTRIBUTIONS**

Parametric sensitivity mathematical models have been developed to analyze and characterize the dynamic behavior of actuating induction motors with solid rotors, which are integral components in advanced control systems [21]. These models provide valuable insights into how variations in specific parameters influence the motor's performance during dynamic operations. Complementing these theoretical models, specialized algorithms and computational tools have been created to accurately calculate the transient parametric sensitivity of these motors. These tools enable engineers and researchers to evaluate the impact of selected parameters, such as rotor resistance, stator inductance, or load conditions, on the motor's transient behavior, facilitating optimized design and control strategies.

# **3.EXPERIMENTAL RESULTS DISCUSSIONS**

We present the fundamental theoretical propositions derived from the analysis of references, serving as the foundation for the development of mathematical models capturing the parametric sensitivities inherent in actuating induction motors featuring solid rotors within complex control systems[22,23]. The provided content includes equations governing the electromagnetic field in both nonlinear movable and immovable media. Expressions delineating the spatial and temporal discretization of these electromagnetic field equations are delineated. Furthermore, a systematic framework for solving the resulting differential equations is established with respect to the vector potential of the electromagnetic field [23,24,25]. The expression for the quasi-stationary electromagnetic field equation within a movable, nonlinear, isotropic medium is provided as follows:

$$
\frac{\partial \mathbf{A}}{\partial t} = -\frac{1}{\gamma} \nabla \times (\mathbf{v} \nabla \times \mathbf{A}) + \mathbf{v} \times \nabla \times \mathbf{A}, \quad (1)
$$

where  $A -$  is the vector potential of the electromagnetic field;  $\bf{v}$  - is the velocity vector;  $\bf{v}$  - is the static medium relativity;  $\gamma$  - is the static conductivity;  $\nabla$  - is the Hamiltonian operator; *t* - is the time. The vectors of electric intensity **E**, magnetic **H** fields and magnetic induction **B** are found by the spatial-time distribution of the vector **A**.

$$
\mathbf{B} = \nabla \times \mathbf{A}; \quad \mathbf{H} = v\mathbf{B}; \quad \mathbf{E} = -\partial \mathbf{A} / dt. \quad (2)
$$

The examination pertains to spatial predicaments within a cylindrical system featuring two dimensions. In this context, the vector potential is characterized by a singular component exclusively tethered to the axis coordinates. The analysis is conducted within the confines of the crosssectional plane of the rotor body, wherein the axial component of the vector potential is oriented orthogonally to the plane of the field.

Directing **A** in the axial coordinate  $A = X_0A$ , the equation (1) assumes the form

$$
\frac{\partial A}{\partial t} = \frac{1}{\gamma} \left( v \frac{\partial^2 A}{\partial r^2} + \frac{v}{r^2} \frac{\partial^2 A}{\partial \alpha^2} + \left( \frac{v}{r} + \frac{\partial v}{\partial r} \right) \frac{\partial A}{\partial r} + \frac{1}{r^2} \right)
$$
(3)

Where  $\omega$  is the angular velocity of rotor rotation; r,  $\alpha$  are the radial and angular coordinates.

The expressions (2) disintegrate into three differential equations

$$
B_r = \frac{1}{r} \frac{\partial A}{\partial \alpha}; \quad B_\alpha = -\frac{\partial A}{\partial r}; \quad E = -\frac{\partial A}{\partial t}, \quad (4)
$$

Where  $B_r$ ,  $B_\alpha$  are the radial and the angular components of magnetic induction vector; E is the axial components of electric field intensity vector.

The expressions (3) and (4) are the basis of electromagnetic process investigation in rotor massive.

The theoretical framework governing a motor equipped with a three-phase stator fundamentally aligns with that of a motor featuring a two-phase stator (Fig. 1). Given the pre localization of stator currents within the windings of the activating induction motor and the use of laminated iron for the core, eddy currents are effectively eliminated. Consequently, the stator circuit is characterized employing methodologies derived from the theory of nonlinear electromagnetic circuits. The presence of eddy currents in the rotor body introduces a distinctive set of considerations, necessitating their description through the application of methods rooted in the theory of quasistationary electromagnetic fields within nonlinear isotropic mediums[26].

Two-phase stator of actuating induction motor contains two asymmetric windings, located at the right angle. One of them is called the excitation winding (e), the other one is called the control winding (c). Differential equations of both windings are

$$
\frac{d\Psi_e}{dt} = u_e - r_e i_e; \quad \frac{d\Psi_c}{dt} = u_c - r_c i_c, \tag{5}
$$



 **Fig.1**. Illustration depicting the circuits of an operating induction motor, receiving power from two distinct sources: one for excitation and the other for control.

Where  $\Psi_e$  and  $\Psi_c$  are complete magnetic-flux linkages;  $i_e$ and  $i_c$  are the currents;  $u_e$  and  $u_c$  are voltages;  $r_e$  and  $r_c$  are resistances. Excitation and control winding voltages are considered as the preset time functions  $u_e(t)$  and  $u_c(t)$ formed in the control of the meter system.

The currents are found by the expressions obtained, provided that dissipation inductivities are constant

$$
\frac{\partial \mathbf{v}}{\partial \mathbf{g}} \frac{\partial \mathbf{A}}{\partial \mathbf{g}} \mathbf{w}_e (\Psi_e \partial \mathbf{g}_e); \quad i_c = \alpha_c (\Psi_c - w_c \Phi_c), \quad (6)
$$
\nWhere  $\Phi_e$ -arde  $\Phi_e$ -arce the principal magnetic fluxes;  $\alpha_e$  and  $\partial \mathbf{g}_c$  are the reverse.

\ngence  $\frac{\partial \mathbf{g}}{\partial \mathbf{g}}$  are the right-hand (or right) and  $\mathbf{g}_c$  are the numbers of winding turns.

The primary fluxes in the winding are determined based on the magnitudes of the radial component of the magnetic induction vector at the rotor's surface.

$$
\Phi_{e} = c \int_{-\pi/2}^{\pi/2} B_{r}(R, \alpha) \cos \alpha d\alpha; \qquad \Phi_{c} = c \int_{-\pi/2}^{\pi/2} B_{r}(R, \alpha) \sin \alpha d\alpha,
$$
\n(7)

Where  $c$  is the constant constructive factor,  $\alpha$  is the angle coordinate.

 $B_r$  calculation is connected with integration of the vector potential equations (3) with further use of differential dependencies (4).

The boundaries of integration (3) is within  $R_1 \le r \le R$ ,  $-\pi/2$  $\leq \alpha \leq \pi/2$ .

Boundary conditions on the external  $(r=R)$  and the internal  $(r=R_1)$  rotor surfaces are found by another expression (4)

$$
\frac{\partial A}{\partial r}\Big|_{r=R} = -B_{\alpha}(R,\alpha); \frac{\partial A}{\partial r}\Big|_{r=R} = -B_{\alpha}(R,\alpha); \quad (8)
$$

Boundary conditions along the integration boundary radii are established based on the field interval.

$$
A(r,\pi)=-A(r,0). \hspace{1.5cm} (9)
$$

To find the angle vector component of the magnetic induction  $B_{\alpha}(R, \alpha)$  on the surface of solid rotor by the law of total current, the author notes the system of nonlinear algebraic equations

$$
H_{\alpha}(R,\alpha) - vB_{\alpha}(R,\alpha) = 0, \qquad (10)
$$

For

$$
H_{\alpha}(R,\alpha) = \frac{1}{R}
$$
  

$$
\left(\rho_{m}\Phi_{e} - \frac{2w_{e}}{\pi \rho_{0}}i_{e}\right)\sin \alpha - \left(\rho_{m}\Phi_{c} - \frac{2w_{c}}{\pi \rho_{0}}i_{c}\right)\cos \alpha.
$$
 (11)

where  $w_e$  and  $w_c$  are the numbers of stator winding turns;  $p_m$  is the statically magnetized stator resistance of circuit and air gap

$$
\mathsf{P}_m = \varphi_m(\Phi_m) / \Phi_m = \mathsf{P}_m(\Phi_m). \tag{12}
$$

The order of the nonlinear algebraic equation system (Equation 10) is dictated by the quantity of spatial grid nodes present on the surface of the rotor. In order to address the boundary problem encompassing equations (3), (4), and (8) to (11), it is imperative to employ the system of ordinary differential equations derived from the discretization of equation (3).

$$
\frac{dA_{i,k}}{dt} = a_{i,k}A_{i-1,k} + b_{i,k}A_{i+1,k} + c_{i,k}A_{i,k-1} + d_{i,k}A_{i,k-1} + g_{i,k}A_{i,k},
$$
\n(13)

*dA*

where  $i$  and  $k$  are the ordinal numbers of grid nodes associated with the radius and the angle;  $a_{i,i}$ ,  $b_{i,k}$ ,  $c_{i,k}$ ,  $d_{i,k}$ and *gi,k* are the approximation coefficients.

The set of ordinary differential equations (Equations 5 and 3) is augmented with equations describing the motion dynamics. LaGrange's equations are utilized to derive comprehensive information from the motion equations, supplying the control system with essential parameters such as the rotor rotation angle and angular velocity.

$$
\frac{d\omega}{dt} = \frac{\rho_0}{J} (M_E - B\omega - C\gamma - M(\omega)), \frac{d\gamma}{dt} = \omega, \quad (14)
$$

Where  $M_e$  is the electromagnetic moment;  $M(\omega)$  is the moment of resistance; J is the addering moment of inertia;  $\rho_0$  is the number of pairs of magnetic poles; *B* is the matrix of dissipation; *C* is the matrix of coefficients of stiffness of elastic connections.

Considering the system of motor as absolutely tight, the first equation (14) is simplified

$$
\frac{d\omega}{dt}=\frac{\rho_0}{J}\left(M_E-M(\omega)\right). \qquad (15)
$$

The initiation of electromagnetic torque  $M(\omega)$  serves as an input mechanical signal within the control system for the actuating motor. This function is regarded as a predefined parameter, derived from the equations governing the mechanical subsystem of the control object. System of the differential equations (5), (13) and (14) is subject to joint integration.

In Fig. 2, it is observed that elevating the electrical conductivity of the rotor material leads to an augmentation in transient response. Subsequently, sensitivity models of the actuating induction motor are examined.



**Fig. 2.** Graphs depicting the rotor rotation velocity during the operation of an unloaded induction motor under various electrical conductivity conditions. of rotor material  $\gamma = 0.25$ . 10<sup>7</sup> Cm/m (curve 1),  $\gamma = 0.5 \cdot 10^7$  Cm/m (curve 2),  $\gamma = 1 \cdot 10^7$ Cm/m

The structure of the column matrix representing the unknown is then addressed.

$$
X = (\Psi_e + \Psi_c, \omega, A_0)_t, \qquad (16)
$$

where  $A_0$  is the column of discrete values of vector potential in a set of nodes of a space grid.

The column of constant parameters is represented as

$$
\lambda = (\lambda_1, \lambda_2, \dots, \lambda_n)_t, \tag{17}
$$

Then the matrix of parametric sensitivities is found as Fresh derivative

$$
\Phi = \frac{\partial X}{\partial \lambda},\tag{18}
$$

The equation (18) can be given as (19)

$$
\Phi = (\chi_e, \chi_c, \alpha, \nu)_t, \tag{19}
$$

Where  $\lambda_e$ ,  $\lambda_c$ ,  $\alpha$  and  $\nu$  are the submatrices of parametric sensitivities

$$
\chi_e = \frac{\partial \Psi_e}{\partial \lambda}; \quad \chi_c = \frac{\partial \Psi_c}{\partial \lambda}; \quad \alpha = \frac{\partial \omega}{\partial \lambda}; \quad v = \frac{\partial A_0}{\partial \lambda}
$$

 Upon obtaining the matrix encompassing parametric sensitivities of the primary variables, it becomes possible to ascertain the corresponding sensitivities for the remaining variables. The parametric current sensitivities for the stator winding are delineated as follows:

$$
\xi_{\mathbf{e}} = \frac{\partial i_{\mathbf{e}}}{\partial \lambda}; \quad \xi_{\mathbf{c}} = \frac{\partial i_{\mathbf{c}}}{\partial \lambda};\tag{21}
$$

$$
\xi_e = \alpha_e \left( \chi_e - \omega_e \frac{\partial \Phi_e}{\partial \lambda} - \frac{\partial \omega_e}{\partial \lambda} \Phi_e \right) +
$$
  
+  $\frac{\partial \alpha_s}{\partial \lambda} (\Psi_e - \omega_e \Phi_e);$   

$$
\xi_c = \alpha_c \left( \chi_c - \omega_c \frac{\partial \Phi_c}{\partial \lambda} - \frac{\partial \omega_c}{\partial \lambda} \right) + \frac{\partial \alpha_s}{\partial \lambda} (\Psi_c - \omega_c \Phi_c)
$$
(23)

Variation equation for  $\chi_e$  and  $\chi_c$  is received, differentiating  $(5)$  on  $\lambda$ , as

$$
\frac{d\chi_e}{dt} = \frac{\partial u_e}{\partial \lambda} - \frac{\partial r_e}{\partial \lambda} i_e - r_{\text{esc}}^{\xi},
$$

$$
\frac{d\chi_c}{dt} = \frac{\partial u_c}{\partial \lambda} - \frac{\partial r_c}{\partial \lambda} i_c - r_{\text{esc}}^{\xi}.
$$
 (24)

It is a complicated thing to receive variation equation for A that is why  $(3)$  is given as  $(25)$ 

$$
\frac{\partial A}{\partial t} = \frac{1}{\gamma} \left( \frac{\partial H_r}{\partial \alpha} - \frac{\partial H_\alpha}{\partial r} - \frac{H_\alpha}{r} \right) - \omega B_r \tag{25}
$$

or as a discretized one

$$
\frac{dA_{i,k}}{dt} = a(H_{r_{i,k-1}} - H_{r_{i,k+1}}) + b(H_{\alpha_{i-1,k}} - H_{r_{i+1,k}}) ++ cH_{\alpha_{i,k}} - d\omega B_{r_{i,k}} + f_{r_{i,k}}
$$
\n(26)

Differentiating (26) on  $\lambda$ , the unknown variation submatrix equation (27) of parametric sensitivity is received

$$
\frac{d\mathbf{v}_{i,k}}{dt} = \mathbf{a}(\pi_{r_{i,k-1}} - \pi_{r_{i,k+1}}) + \mathbf{b}(\pi_{\alpha_{i-1,k}} - \pi_{\alpha_{i+1,k}}) +
$$
\n
$$
+ c_{\gamma_{\alpha_{i,k}}} - \mathbf{d}(\alpha_i B_{r_{i,k}} + \omega \beta_{r_{i,k}}) + \frac{\partial f_{r_{i,k}}}{\partial \lambda} + \frac{\partial \mathbf{a}}{\partial \lambda} (H_{r_{i,k-1}} - H_{r_{i,k+1}}) + \frac{\partial \mathbf{b}}{\partial \lambda} (H_{\alpha_{i-1,k}} - H_{\alpha_{i+1,k}}) + \frac{\partial \mathbf{c}}{\partial \lambda} H_{\alpha_{i,k}} - \frac{\partial \mathbf{d}}{\partial \lambda} \omega B_{r_{i,k}},
$$
\n(27)

Where  $\pi_r$ ,  $\pi_\alpha$ ,  $\beta_r$  and  $\beta_\alpha$  are the submatrices of the parametric sensitivity of magnetic field vectors

$$
\pi_r = \frac{\partial H_r}{\partial \lambda}; \ \pi_\alpha = \frac{\partial H_\alpha}{\partial \lambda}; \ \beta_r = \frac{\partial B_r}{\partial \lambda}; \ \beta_\alpha = \frac{\partial B_\alpha}{\partial \lambda} \tag{28}
$$

for that

$$
H_{i,k} = \mathbf{v}_{i,k}' B_{i,k},
$$
  

$$
\gamma_{i,k} = N_{i,k} \beta_{i,k} + \frac{\partial \mathbf{v}_{i,k}'}{\partial \lambda} B_{i,k},
$$
 (29)

In this case  $N_{i,k}$  is the local submatrix of differential ferromagnetic reluctivity in the node of *i,k*

$$
N_{i,k} = \frac{\begin{vmatrix} v'_{i,k} - v'_{i,k} \end{vmatrix} B_{i_{i,k}}^2}{\begin{vmatrix} v''_{i,k} - v'_{i,k} \end{vmatrix} B_{i_{i,k}} \frac{B_{i_{i,k}}^2}{B_{i_{i,k}}^2}} \frac{\begin{vmatrix} (v''_{i,k} - v'_{i,k}) B_{i_{i,k}} B_{\alpha_{i,k}} \end{vmatrix}}{B_{i,k}^2} \frac{\begin{vmatrix} b''_{i,k} - v'_{i,k} \end{vmatrix} B_{\alpha_{i,k}}}{\begin{vmatrix} v''_{i,k} - v'_{i,k} \end{vmatrix} B_{\alpha_{i,k}}^2}}{\begin{vmatrix} b''_{i,k} - v'_{i,k} \end{vmatrix} B_{i_{i,k}}^2} \frac{\begin{vmatrix} b''_{i,k} - b''_{i,k} \end{vmatrix} B_{\alpha_{i,k}}^2}{\begin{vmatrix} b''_{i,k} - b''_{i,k} \end{vmatrix} B_{\alpha_{i,k}}^2} \frac{\begin{vmatrix} b''_{i,k} - b''_{i,k} \end{vmatrix} B_{\alpha_{i,k}}^2}{\begin{vmatrix} b''_{i,k} - b''_{i,k} \end{vmatrix} B_{\alpha_{i,k}}^2} \frac{\begin{vmatrix} b''_{i,k} - b''_{i,k} \end{vmatrix} B_{\alpha_{i,k}}^2}{\begin{vmatrix} b''_{i,k} - b''_{i,k} \end{vmatrix} B_{\alpha_{i,k}}^2} \frac{\begin{vmatrix} b''_{i,k} - b''_{i,k} \end{vmatrix} B_{\alpha_{i,k}}^2}{\begin{vmatrix} b''_{i,k} - b''_{i,k} \end{vmatrix} B_{\alpha_{i,k}}^2} \frac{\begin{vmatrix} b''_{i,k} - b''_{i,k} \end{vmatrix} B_{\alpha_{i,k}}^2}{\begin{vmatrix} b''_{i,k} - b''_{i,k} \end{vmatrix} B_{\alpha_{i,k}}^2} \frac{\begin{vmatrix} b''_{i,k} - b''_{i,k} \end{vmatrix} B_{\alpha_{i,k}}^2}{\begin{vmatrix} b''_{i,k} - b''_{i,k} \end{vmatrix} B_{\alpha_{i,k}}^2} \frac{\begin{vmatrix} b''_{i,k} - b''_{i,k} \end{vmatrix} B_{\alpha_{i,k}}^2}{\begin{vmatrix} b''_{i,k} - b''_{i,k} \
$$

At this *v*, *v*" are the statically and differential reluctivity of ferromagnetic

$$
\mathsf{V}=\frac{\mathsf{H}(B)}{B};\;\;\mathsf{V}^{\prime}=\frac{\mathsf{d}\mathsf{H}(B)}{\mathsf{d}B},\;\;B_{i,k}=\sqrt{B_{r_{i,k}}^2+B_{\alpha_{i,k}}^2}.
$$
\n(31)

Submatrices of parametric sensitivities of magnetic flux windings are found in the following way

$$
\chi'_e = \frac{\partial \Phi_e}{\partial \lambda} = \frac{c_0 \pi}{3n} (\beta_{r0} + 4\beta_{r1} \cos \alpha_1 + 2\beta_{r2} \cos \alpha_2 + ...
$$
  
+ 4\beta\_{rn-1} \cos \alpha\_{n-1} - \beta\_{rn}) + \frac{1}{c\_0} \frac{\partial c\_0}{\partial \lambda} \Phi\_e. (32)  

$$
\chi'_c = \frac{\partial \Phi_c}{\partial \lambda} = \frac{c_0 \pi}{2} (4\beta_{r1} \sin \alpha_1 + 2\beta_{r2} \sin \alpha_2 + ...
$$

$$
\chi_c = \frac{1}{\partial \lambda} = \frac{1}{3n} \left( 4p_{r1} \sin \alpha_1 + 2p_{r2} \sin \alpha_2 + \dots \right)
$$

$$
+ 4\beta_{rn-1} \sin \alpha_{n-1} + \frac{1}{c_0} \frac{\partial c_0}{\partial \lambda} \Phi_c.
$$
(33)

Sensitivity of mechanical variables is received by differentiating on  $\lambda$  the equation (16)

$$
\frac{d\alpha}{dt} = \frac{1}{J} \left( p_0 \left( \frac{3}{2} p_0 \chi_e i_e + \Psi_e \xi_e - \chi_e i_e - \Psi_e \xi_e - \frac{\partial M(\omega)}{\partial \omega} \right) \alpha - \frac{\partial J}{\partial \lambda} \frac{d\omega}{dt} \right)
$$
(34)

An algorithm and computer programs for calculating transient parametric sensitivity in actuating induction motors with solid ferromagnetic rotors under constant parameters have been devised. These tools enable the computation of several transient parameters associated with the rotor body material, derived from the input signals of constant parameters.

Calculation of space distribution of parametric sensitivity of electromagnetic field vector potential to rotor material electrical conductivity ( $\gamma = 0.5 \cdot 10^7$  Cm/m) at the fixed time is shown in Fig. 3.



**Fig. 3.** Spatial distribution of the parametric sensitivity of vector potential to electrical conductivity on pole pitch of the rotor crosssection in transient regime without loading at the moment of time t=3s at the electrical conductivity  $\gamma = 0.5 \cdot 10^7$  Cm/m.

Observing Fig. 4 reveals that the parametric sensitivity of rotor rotation velocity to rotor material conductivity is most pronounced during the time interval from motor startup to the attainment of the rated rotational frequency, diminishing as time progresses.

Examining Fig.5 highlights that the peak parametric sensitivity of the primary magnetic flux in the excitation winding to rotor material conductivity aligns with the moment when the actuating induction motor reaches its rated rotation.



**Fig. 4.** Calculation curve of parametric sensitivity of angular winding velocity to rotor electrical conductivity of actuating induction motor in transient unloading start at the electrical conductivity  $\gamma = 0.25 \cdot 10^7$  Cm/m



**Fig. 5.** Transient unloading of an actuating induction motor initiates a calculation curve representing the parametric sensitivity of the main magnetic flux in the excitation winding concerning the electrical conductivity of the rotor material  $\gamma = 0.25 \cdot 10^7$  Cm/m.

# **4. CONCLUSION**

The optimal mathematical model, derived from the integration of electromagnetic circuit theories and electromagnetic field principles, is determined to be accurate, efficient, and expeditious. This model is wellsuited for describing elements within a control system.

By employing the method of finite difference and the implicit principle is recommended for the spatial discretization of the differential equations governing actuating induction motors with solid hollow ferromagnetic rotors, as well as their first-order variation equations of parametric sensitivities.

Additionally the provided computer program for calculating parametric sensitivity is applicable to optimization problems related to the geometric dimensions and materials used in one- and two-phase rotors. Moreover the analysis of computer simulation results underscores the significant impact of the electrical conductivity of the rotor body on electromagnetic motor indices. This mathematical

confirmation supports the rationale and practicality of advancing the design concept towards motors with multilayer rotors.

Finally, the implementations detailed in the paper illustrate that the mathematical techniques for constructing parametric sensitivity models, originally grounded in a unified theory of nonlinear ordinary differential equations, are highly versatile. These methods can be successfully extended and adapted to more complex scenarios, including those involving partial differential equations (PDEs) and systems combining both ordinary differential equations (ODEs) and partial differential equations. This adaptability underscores the robustness of the underlying theory, enabling it to address a broader spectrum of dynamic systems. By incorporating these methods, researchers can systematically analyze the sensitivity of systems governed by spatial and temporal dynamics, thus providing valuable insights into their behavior under varying parameter conditions. These advancements pave the way for more comprehensive modeling approaches in fields such as control systems, fluid dynamics, and thermomechanical processes.

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