Volume:03 Issue:06 Pages:1430-1437 (2012) ISSN : 0975-0290.

Binary Decision Diagrams and Its Variable Ordering for Disjoint Network

Manoj Singhal
Associate professor, Department of Information Technology,
Accurate Institute of Management & Technology, Greater Noida (U.P.), India,
Email: manoj_singhal@rediffmail.com
Girish Sharma
Associate Professor, Department of Computer Applications, Bhai Parmanand Institute of Business Studies, New Delhi
Email: gkps123@gmail.com
R. K. Chauhan
Professor, Département of Computer Science and Applications, Kurukshetra University, Kurukshetra
Email : rkchauhankuk@gmail.com

ABSTRACT
We know that binary decision diagram is a data structure that is used to store a Boolean function. They are used to find out the terminal reliability of a computer communication network. To generate the binary decision diagram of a given computer communication network, we need to order the edges of the given computer communication network because the size of the binary decision diagram is dependent on the ordering of the variables (edges). There are three types of variable ordering; optimal, good and bad ordering. Optimal ordering are those ordering which generate minimum size binary decision diagram. In this paper we have shown that if a directed computer communication network has m disjoints min-paths then m! optimal variable orderings exist to generate the binary decision diagrams of the given computer communication network.

Keywords - Binary Decision Diagrams (BDD), Computer communication Network (CNN), Directed Acyclic Graph (DAG), Modified Binary Decision Diagrams (MBDD)

Date of Submission: January 23, 2012 Date of Acceptance: March 29, 2012

I. INTRODUCTION

Akers [1] first introduced binary decision diagram (BDD) to represent Boolean functions i.e. a BDD is a data structure used to represent a Boolean Function. Bryant [2] popularized the use of BDD by introducing a set of algorithms for efficient construction and manipulation of BDD structure. The BDD structure provides compact representations of Boolean expressions. The authors [3] have shown a method to minimize Boolean expression with sum of disjoint product functions by using BDD. BDD are used to work out the terminal reliability of the links. Madre and coudert found BDD usefulness in reliability analysis which was further extended by Rauzy [4, 5]. They are specially used to assess fault trees in system analysis. In the network reliability framework, Sekine & Imai [6], and Trivedi [7] have shown how to functionally construct the corresponding BDD. One method to compute the network reliability has shown by Roberta [8]. An alternate approach to compute the network reliability by using BDD was presented by Singhal [9]. BDD are also used to find out the network reliability with unreliable nodes. It means the network has imperfect nodes as well as imperfect links. A modified combined method to compute terminal- pair reliability with unreliable nodes was presented by Zhong [10]. One efficient and exact method was shown to compute the network reliability with imperfect vertices by Lin [11]. One other method was shown by Xing with imperfect coverage and common cause failure [12]. Network reliability calculation can be improved by augmented BDD. This has shown by Herrmann [13]. BDD variable order optimization is a well-studied problem. Over the years, researchers have proposed many techniques to order the variables. BDD methodology is the most recent approach to improve Boolean reliability models assessment. The final size of the BDD after applying the three reduction rules is the ultimate technique to reduce the size of the BDD. Several variable ordering strategies have been proposed within the literature. All of them have focused to generate the minimum size BDD. The final size of a BDD heavily depends on the selection of variable ordering. From the beginning of the BDD, this problem is well-known. Bryant described the need for a good ordering to consolidate the technology in 1980. Consequently, it has become a crucial point of this methodology. There is still scope to improve and extend the current ordering methodologies. Due to its combinatorial nature, finding the optimal ordering is a NP-complete problem. Therefore, it is intractable computationally. This was shown by Bollig & Wegener [14] in 1996. In order to find good orderings to generate the BDD, researchers have been made to design heuristics which provides good orderings within reasonable time and minimum memory requirements. The authors [15] have shown a new approach to find the various optimal ordering to generate minimum size BDD. The classification of

Int. J. Advanced Networking and Applications 1431
Volume:03 Issue:06 Pages:1430-1437 (2012) ISSN : 0975-0290.

variable ordering strategies can be arranged into two main groups. These are static heuristics and dynamic heuristics. The static heuristic provides an initial variable ordering prior to the BDD construction. These heuristic are based on topological considerations. The dynamic heuristics are used to change the variable ordering during the BDD construction process, and are based on swapping and shifting groups of variables at some points of the computation [16]. The dynamic reordering is very expensive in terms of memory and time. It may help to improve the final result, but without a good initial order, they are not used from the outset in reliability analysis. In the reliability field, static heuristic is widely used due to lesser requirement of memory and time.
There are three types of variable ordering to generate the BDD. This classification is based on the size of the different BDD. These are optimal, good and bad ordering [17]. Optimal ordering are those ordering which generate minimum size BDD. The size of the BDD means the number of non-terminal vertices in the BDD and the number of non-terminal vertices at particular level [18].

This paper is organized as follows. First we will illustrate the preliminaries of BDD in Section 2. In Section 3, we have shown that if a directed computer communication network has m disjoints min-paths then m! optimal variable orderings exist to generate the BDD of the given CCN. Finally we draw some conclusion.

II. BINARY DECISIONS DIAGRAMS

BDD is a data structure used to represent a Boolean Function. A BDD is a directed acyclic graph (DAG) based on the Shannon decomposition. The Shannon decomposition for a Boolean function is defined as follows:

$$f = x.\, f_{x=1} + \overline{x}.\, f_{x=0}$$

where x is one of the decision variables, and f is the Boolean function evaluated at x = i. By using Shannon's decomposition, any Boolean expression can be transformed in to binary tree. Table 1 shows the truth table of a Boolean function f and its corresponding Shannon tree is shown in figure 1.

Table 1

Truth Table of a Boolean Function f			
x_1	x_2	x_3	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

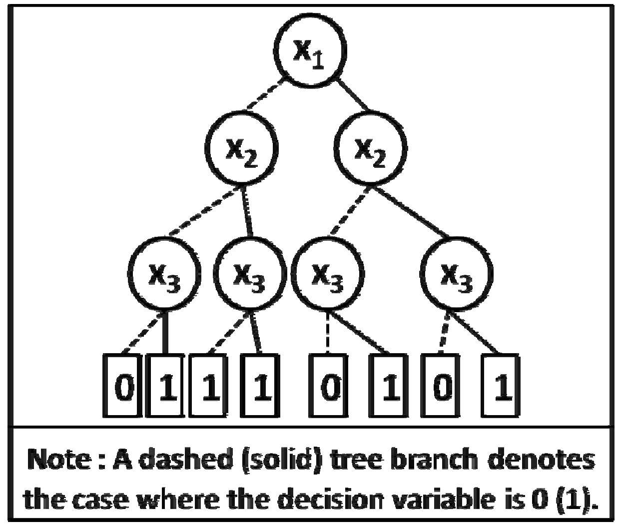


Figure 1: Shannon Tree of a given Boolean Function f

Sink nodes are labelled either with 0, or with 1, representing the two corresponding constant expressions. Each internal node u is labelled with a Boolean variable var(u), and has two out-edges called 0-edge, and 1-edge. The node linked by the 1-edge represents the Boolean expression when $x_i = 1$, i.e. $f_{xi=1}$; while the node linked by the 0-edge represents the Boolean expression when xi = 0, i.e. fxi=0. The two outgoing edges are given by two functions low(u) and high(u).

Indeed, such representation is space consuming. It is possible to shrink by using following three postulates.

Remove Duplicate Terminals : Delete all but one terminal vertex with a given label, and redirect all arcs into the deleted vertices to the remaining one.

Delete Redundant Non Terminals : If non terminal vertices u, and v have var(u) = var(v), low(u) = low(v), and high(u) = high(v), then delete one of the two vertices, and redirect all incoming arcs to the other vertex.

Delete Duplicate tests : If non terminal vertex v has low(v) = high(v), then delete v, and redirect all incoming arcs to low(v).

If we apply all these three rules then the decision tree can be reduced. The related diagram is shown in figure 2.

Volume:03 Issue:06 Pages:1430-1437 (2012) ISSN : 0975-0290.

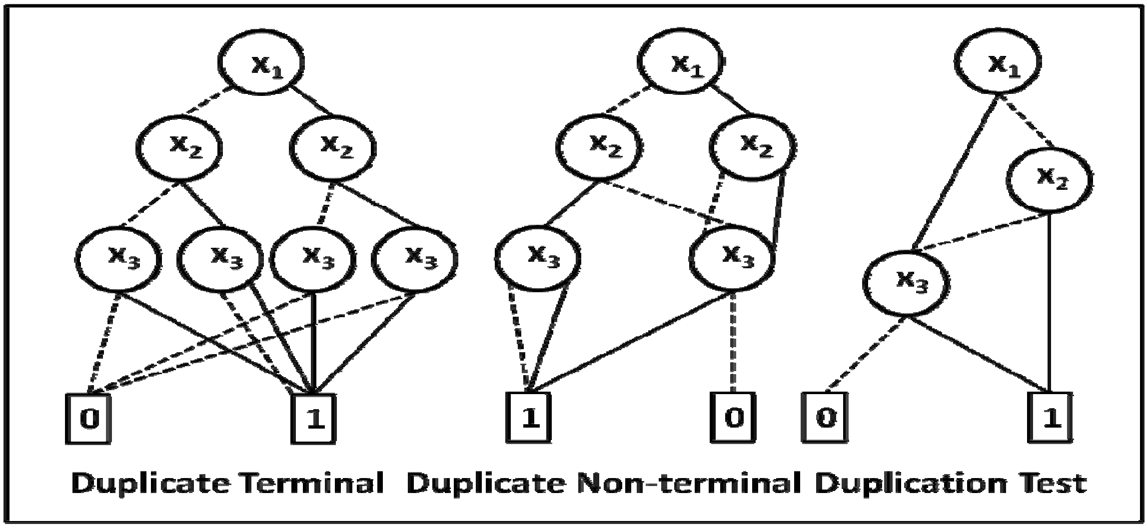


Figure 2: Shrinking Process of the above Decision Tree

2.1 Dual Binary Decision Diagrams:
If two or more BDD have the same size and representing the same Boolean function, then these BDD are known as Dual BDD, because they are Dual of each other.

2.1 Modified Binary Decision Diagram:
The modified binary decision diagram (MBDD) is a binary decision diagram which is either dual BDD or the smaller size BDD. It is necessary that both the BDD must represent the same Boolean function [19].

III PROOF:
If all the min-paths from source to sink in a CCN are disjoint then such type of CCN is known as disjoint network. Now let us suppose that there is only one min-path from source to sink. It means that at least one edge is there between source and sink. There may be more than one edge between source and sink. Few networks are shown in figure 3 consisting of only one min-path from source to sink.

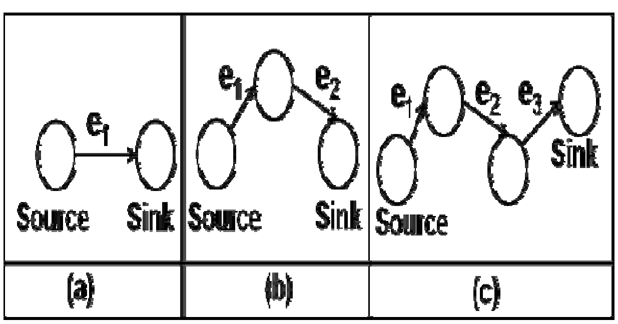


Figure 3: Example Network Containing One min-path

The only possible orderings for figure 3(a) is e_1, for figure 3(b) is $e_1 \prec e_2$ and for figure 3(c) is $e_1 \prec e_2 \prec e_3$.
The respective binary decision diagrams are shown in figure 4.

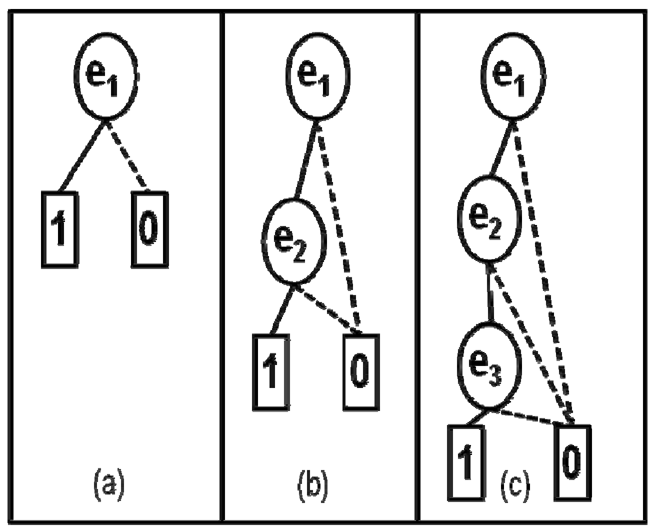


Figure 4: BDD of networks given in figure 3

Now let us suppose that there are two disjoint min-paths from source to sink. The networks containing two min-paths can be represented in the form of graphs shown in figure 5.

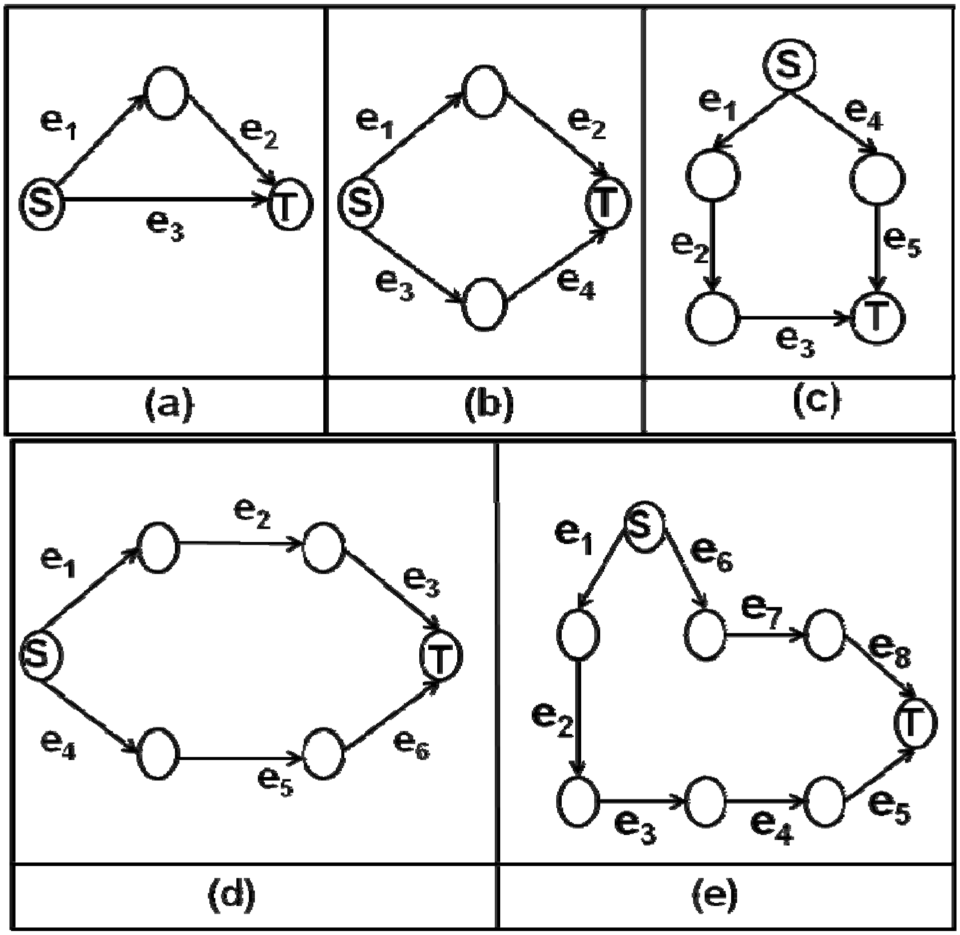


Figure 5: Example Networks containing Two min-paths.

Since there are two min-paths from source to sink, so there must exist 2! = 2 optimal variable ordering to generate the BDD of the given directed networks shown in figure 5. The respective orderings are as follows:

(a) $e_1 \prec e_2 \prec e_3$ and $e_3 \prec e_1 \prec e_2$

(b) $e_1 \prec e_2 \prec e_3 \prec e_4$ and $e_3 \prec e_4 \prec e_1 \prec e_2$

(c) $e_1 \prec e_2 \prec e_3 \prec e_4 \prec e_5$ and $e_4 \prec e_5 \prec e_1 \prec e_2 \prec e_3$

(d) $e_1 \prec e_2 \prec e_3 \prec e_4 \prec e_5 \prec e_6$ and $e_4 \prec e_5 \prec e_6 \prec e_1 \prec e_2 \prec e_3$

(e) $e_1 \prec e_2 \prec e_3 \prec e_4 \prec e_5 \prec e_6 \prec e_7 \prec e_8$ and $e_6 \prec e_7 \prec e_8 \prec e_1 \prec e_2 \prec e_3 \prec e_4 \prec e_5$

The respective BDD are shown in figure 6.

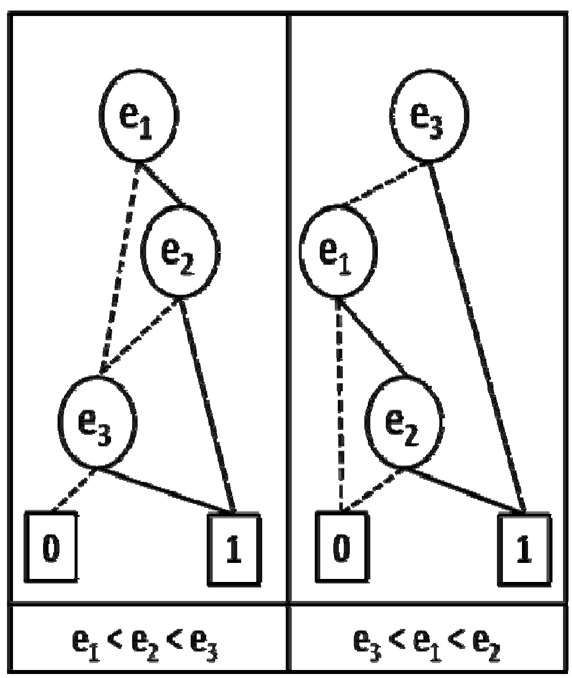


Figure 6(a): BDD of networks given in figure 5(a)

Volume:03 Issue:06 Pages:1430-1437 (2012) ISSN : 0975-0290.

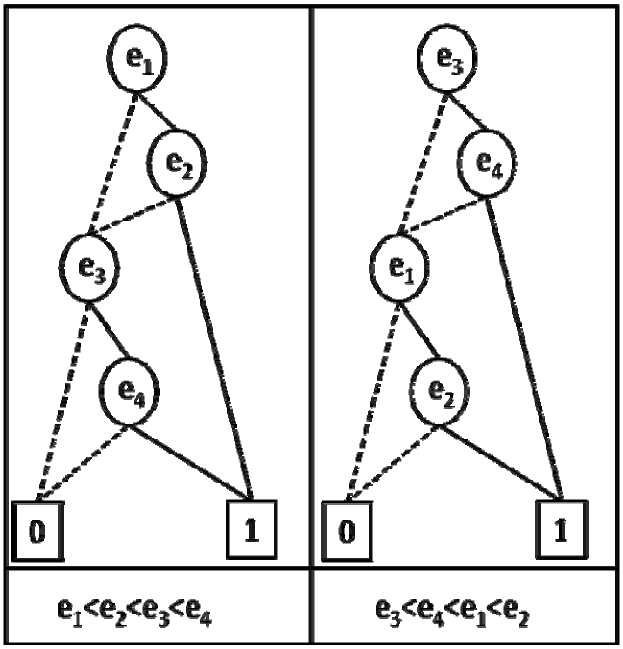


Figure 6(b): BDD of networks given in figure 5(b)

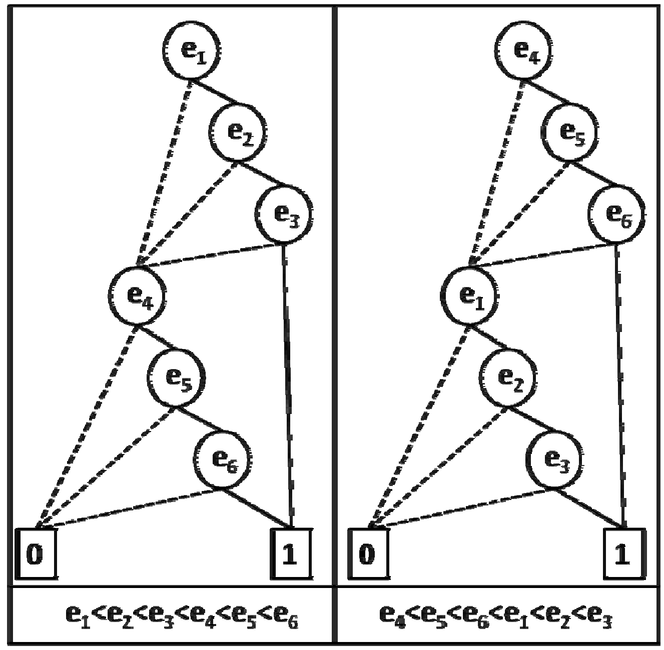


Figure 6(d): BDD of networks given in figure 5(d)

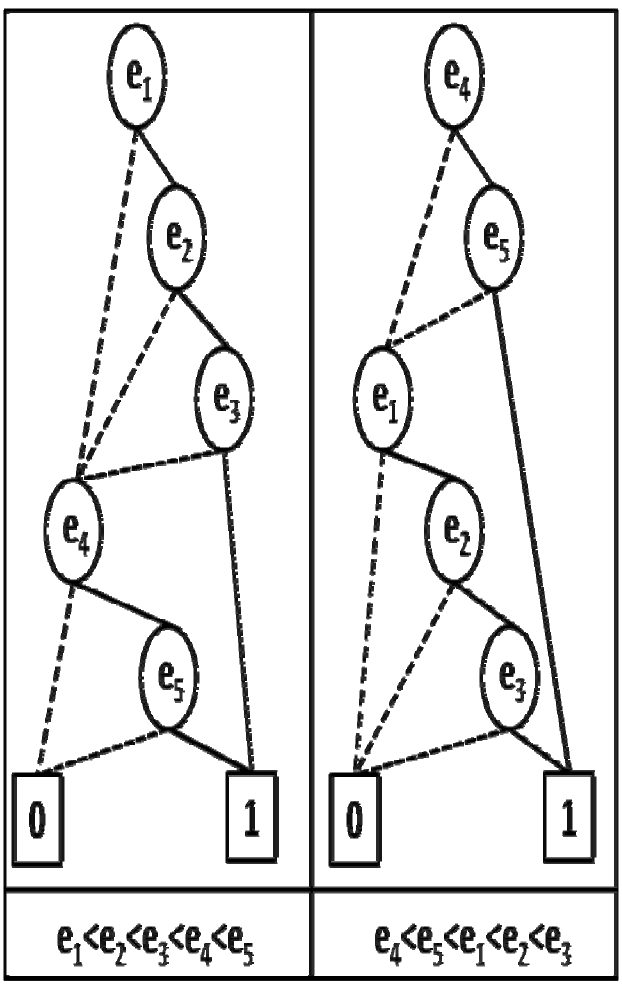


Figure 6(c): BDD of networks given in figure 5(c)

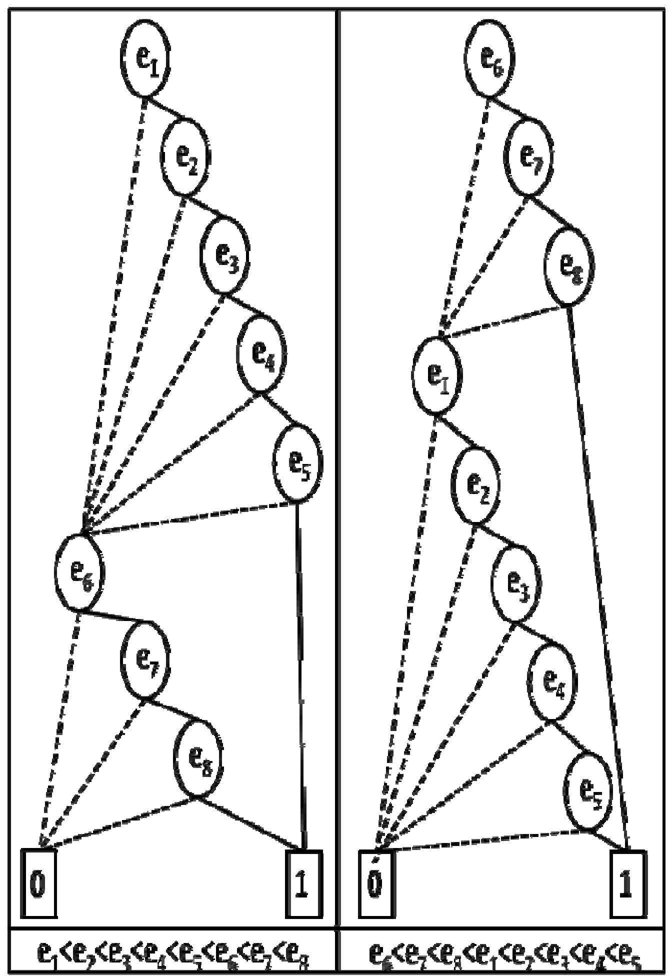


Figure 6(e): BDD of networks given in figure 5(e)

Volume:03 Issue:06 Pages:1430-1437 (2012) ISSN : 0975-0290.

We have shown that there exist 2! =2 optimal variable orderings for the example network shown in figure5 (a), 5(b), 5(c), 5(d) and 5(e) respectively.

Now let us suppose that there exist only three disjoint min-paths in a network as shown in figure 7.

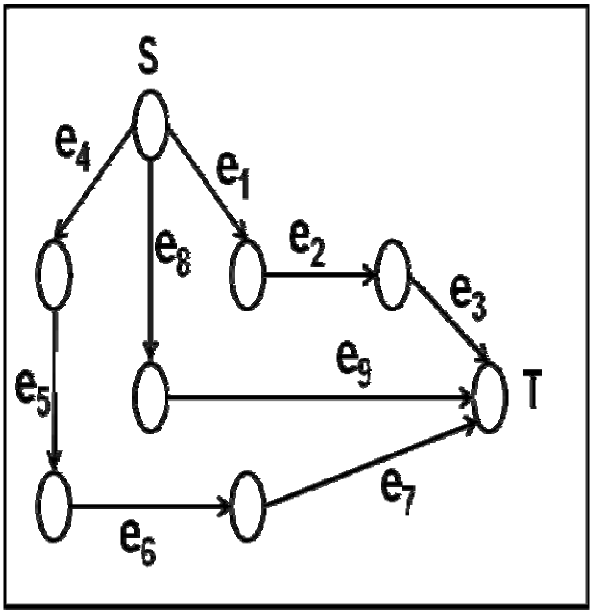
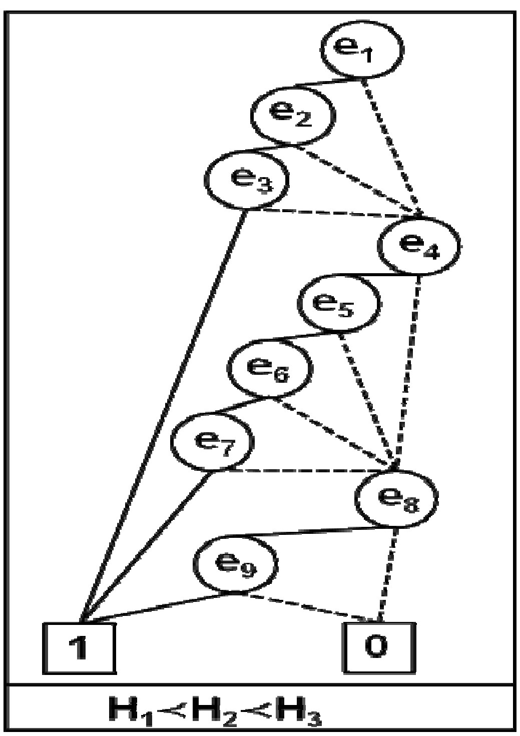
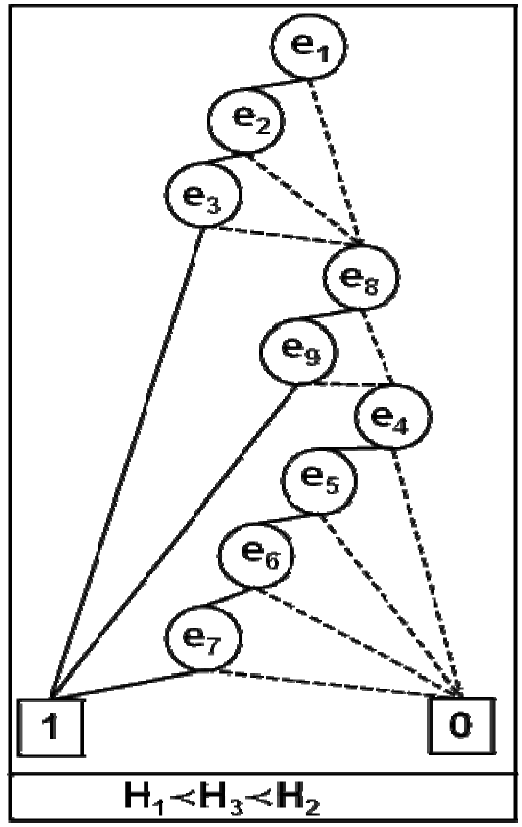
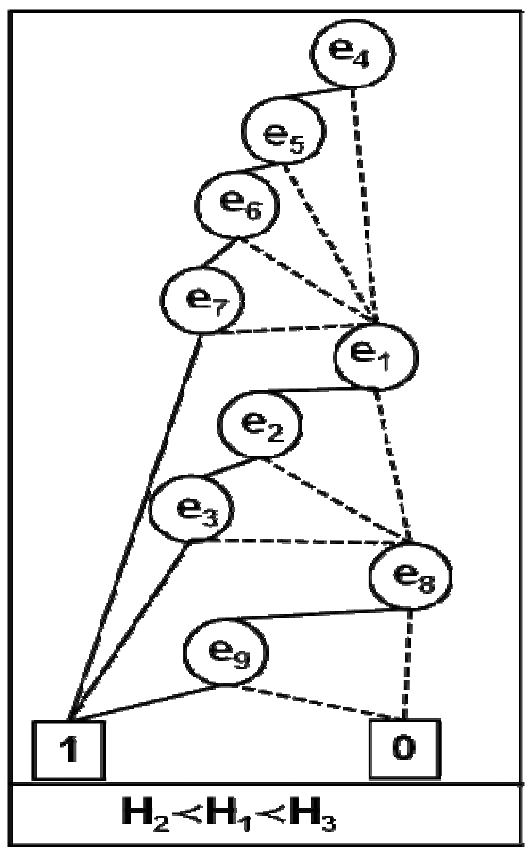


Figure 7: Network having three disjoint min-paths

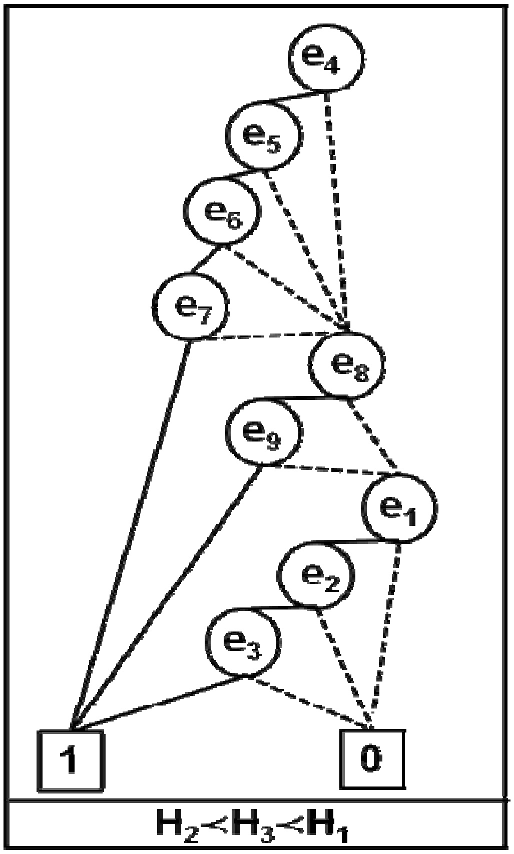
As the network consists three disjoint min-paths say H_1, H_2 and H_3. Here $H_1 = \{e_1, e_2, e_3\}$, $H_2 = \{e_4, e_5, e_6, e_7\}$, $H_3 = \{e_8, e_9\}$. The possible variable orderings are $H_1 \prec H_2 \prec H_3$, $H_1 \prec H_3 \prec H_2$, $H_2 \prec H_1 \prec H_3$, $H_2 \prec H_3 \prec H_1$, $H_3 \prec H_1 \prec H_2$ and $H_3 \prec H_2 \prec H_1$. The BDD are shown in figure 8.

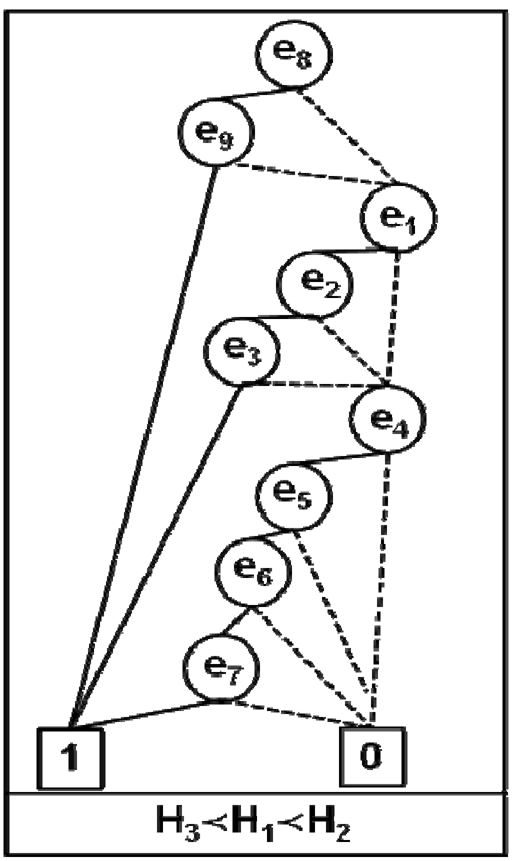






Volume:03 Issue:06 Pages:1430-1437 (2012) ISSN : 0975-0290.





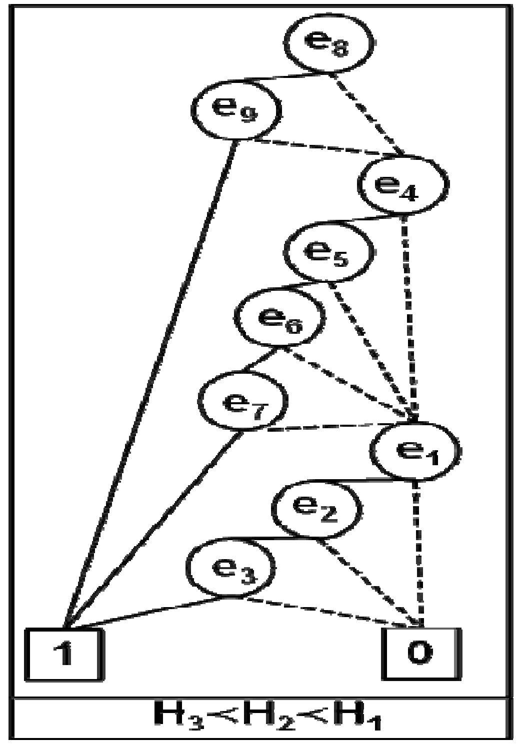


Figure 8: Respective BDD of the network shown in figure 7

In general, if there are m disjoint min-paths from source to sink then m! optimal variable ordering are possible to generate the BDD. To prove this we will consider that there are m disjoint min-paths of a CCN. To find the total number of optimal ordering to generate the BDD of the given CCN which consists of m disjoint min-paths is equal to the number of different ways of filling up m positions by these m disjoint min-paths.

Since any one of these m min-paths can be placed in the first position, so the number of different ways of filling up the first position = m.

When the first position is filled up, the second position can be filled up by any one of the remaining (m – 1) min-paths. So the number of different ways of filling up the second position = (m – 1).

Counting in this way the last position is filled by m – (m – 1) ways i.e. 1 ways.

Hence by fundamental principal of counting, these m positions can be filled up in ways

= m (m – 1) (m – 2) ....{m – (m – 2)}. {m – (m – 1)}.

= m (m – 1) (m – 2) 2.1

= m!

Thus we have shown that, if a directed CCN has m disjoints min-paths then m! optimal variable orderings exist to generate the BDD.

IV Conclusions:

In this paper we have shown that, if a directed CCN has m disjoints min-paths, then m! optimal variable orderings exist to generate the BDD of the given network.

REFERENCES

[1]. B. Akers, "Binary decision diagrams", *IEEE Trans. Computers*, vol. C-27, 1978, 509–516. .

[2]. R. E. Bryant, "Symbolic Boolean manipulation with ordered binary-decision diagrams", *ACM Computing Surveys*, vol. 24, no. 3, 1992, 293–318.

[3]. D. Sensarma, S. Naskar and S. S. Sarma, "On An Optimization Technique using Binary Decision Diagram", *IJCSEA*, Vol.2 No. 1, 2012, 73-86.

[4]. A. Rauzy, "New algorithms for fault tolerant trees analysis", *Reliability Engineering and System Safety*, vol. 5, no. 59,1993, 203–211.

[5]. A. Rauzy, "A new methodology to handle Boolean models with loops", *IEEE Trans. Reliability*, vol. R-52, no. 1, 2003, 96–105.

[6]. H. Imai, K. Sekine, and K. Imai, "Computational investigations of all terminal network reliability via BDDs", *IEICE Transactions on Fundamentals*, vol. E82-A, no. 5, 1999, .714–721.

[7]. X. Zang, H. Sun, and K. S. Trivedi, "A BDD-based algorithm for reliability Graph Analysis", *Technical Report* [Online]. Available: http://www.ee.duke.edu/~hairong/workinduke/relgrap

[8]. Andrea Bobbio, Caterina Ferraris, Roberta Terruggia, "New Challenges in Network Reliability Analysis", *Technical Report*, 2006, 1-8, TR-INF-UNIPMN.

[9]. Manoj Singhal, R. K. Chauhan, , Girish Sharma, "An alternate Approach to Compute the Reliability of a Computer Communication Network using Binary Decision Diagrams", *Communications in Computer and Information Science*, pp. 160-170, IC3 2010, Springer-Verlag Berlin Heidelberg .

[10]. Y. Chen, A. Q. Hu, K. W. Yip, X. Hu, and Z. G. Zhong, "A modified combined method for computing terminal-pair reliability in networks with unreliable nodes," in Proceedings of the 2nd Int'l Conference *on Machine Learning and Cybernetics*, 2003, 2426–2429.

[11]. Sy-Yen Kuo, Fu-Min Yeh, and Hung-Yau Lin, "Efficient and Exact Reliability Evaluation for Networks With Imperfect Vertices", *IEEE Transactions on Reliability*, vol. 56, no. 2, 2007, 288-300.

[12]. Liudong Xing, "An Efficient Binary Decision Diagram Based Approach for Network Reliability and Sensitivity Analysis", *IEEE Trans. On Systems*, Man and Cybernetics, 2008, 105-115.

[13]. J. U. Herrmann, "Improving Reliability Calculation with Augmented Binary Decision Diagrams", IEEE International Conference on *Advanced Information Networking and Applications*, 2010, 328-333.

[14]. Bollig, B. & Wegener, I., "Improving the variable ordering of OBDD is NP-complete", *IEEE Transactions on Software Engineering*, 45(9), 1996: 993-100.

[15]. Manoj Singhal, R. K. Chauhan, Girish Sharma, "A New approach for finding the Various Optimal Variable Ordering to generate the Binary Decision Diagrams (BDD) of a Computer Communication Network" International *Journal of Computer Application*, Vol. 31, No.3, 2011, 1-8.

[16]. R. Rudell, "Dynamic Variable Ordering for Ordered Binary Decision Diagrams," in Proceedings of the International Conference on *Computer Aided Design (ICCAD)*, 1993, 42-47.

[17]. Manoj Singhal, R. K. Chauhan, Girish Sharma, "Network Reliability Computation using Different Binary Decision Diagrams", *International Journal of Distributed and Parallel Systems*, Vol. 1, No. 1, 2010, 82-91.

[18]. Manoj Singhal, R. K. Chauhan, , Girish Sharma, "A New Optimal Approach for evaluating the size of BDD (Binary Decision Diagram) for calculating the Reliability of a CCN (Computer Communication Network)". *International Journal of Advanced Networking and Applications*. Vol. 1, 2010, 230-235.

[19]. Manoj Singhal, R. K. Chauhan, Girish Sharma, "Use of Modified Binary Decision Diagrams in Reliability Evaluation of a Directed Computer Communication Network", The Icfai University *Journal of Computer Sciences*, Vol. III No. 3, 2009, 22-30.

Authors Biography

Mr. Manoj Singhal is currently working as an Associate Professor, Department of Information Technology, Accurate Institute of Management and Technology, Greater Noida (U.P.), India. He has published 10 research papers in International/National Journal/Conference. He has published one book on Discrete Mathematics.

Dr. Girish Sharma is currently working as an Asso. Professor, Department of Computer Application in Bhai Parmanand Institute of Business Studies, New Delhi. He has published more than 15 research papers in International/National journal / Conference. His research interest includes Mathematical Modelling, Distributed Systems and Parallel Architecture.

Prof. (Dr.) R. K. Chauhan is currently working as a Professor, Department of Computer Science and Application in Kurukshetra University, Kurukshetra. He has published more than 70 research papers in International/National Journal/Conference. He has guided 7 Ph.D. His research interest includes Data Mining, DBMS and Algorithm.